Clarifications and Corrections for *An Invitation to Real Analysis*

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Definition 1.4.17 (page 21, clarification) By the axiom of choice, there exists a subset $\mathscr{I}$ of X such that each element of X is equivalent to *precisely one* element of $\mathscr{I}$.

Definition 3.1.4 (page 49, clarification) If x is a real number, we call the product $x \cdot x$ the *square* of x, and denote it by x^2, read *x squared*.

Remark 4.1.2 (page 67, clarification) Generally, assume $(F, +, \cdot, <)$ is an ordered field and $I \subseteq F$. We say I is an *interval* if the following property holds: For all x and y in I and for all z in F, $x < z < y$ implies $z \in I$.

In words, if two elements x and y of F are in I, then so is every element of F between x and y.

Particularly, in an ordered field the empty set is an interval, and every singleton is an interval.

Exercise 5.4.9 (page 116, clarification) A proof by contraposition can be given.

Definition 6.1.6 (page 118, clarification) A real sequence that does not converge to a real number is said to *diverge*.

Exercise 6.4.9 (d) (page 137, clarification) The notation $\mathbf{Q}_{q_n}$ is introduced in Example 4.1.14, pp. 69–70.

Exercise 7.1.10 (page 143, clarification) The sum is taken over positive integers k.

Definition 7.2.10 (page 147, correction) $a_0b^2 \to a_0b_2$.

Exercise 8.4.2 (page 179, clarification) If $x < 0$, leaving the expression $x^{m/n}$ undefined if m and n have a common factor guards against the type of formal error in Example 8.4.10. Really, we define $x^{m/n}$ by reducing the exponent to lowest terms, say m'/n', then checking that n' is odd. Thus, for all real x we define $x^{2/6} := x^{1/3}$, while $x^{6/4} := x^{3/2}$ is undefined if $x < 0$.

Exercise 8.4.14 (page 180, clarification) The task should be read, "Prove f is discontinuous at every point of I."

Exercise 8.6.5 (page 187, clarification) "converges to x_∞" means "converges to a repelling fixed point x_∞."

Exercise 10.4.11 (page 234, clarification) Part of the question is to think about how to differentiate a function defined by an integral without using a fundamental theorem of calculus.

Example 11.3.2 (page 248, clarification) At this stage of the book we must assume p is positive and *rational*. (The results asserted in this section are true for all positive real p, but real exponentiation has not yet been defined.)

Exercise 12.1.12 (page 258, clarification) Here "integer part" means "floor" (Definition 4.3.3, p. 79).

Proposition 12.3.6 (page 266, correction) The formula for the derivative of cosh is true if $x > 1$ (not $x \geq 1$).

Exercise 13.1.4 (page 280, clarification) This exercise refers to π, which is not defined until Section 13.2, but can be done by deleting the reference to $\pi/2$.

Proposition 13.2.10 (page 284, correction) In the proof, "Substitute $t = \sin^2 t$" means "Substitute $t = \sin^2 u$, then replace u by t."

Remark 14.1.2 (page 297, clarification) In the first paragraph, delete the sentence, "Much of complex analysis rests on a choice of imaginary unit."

In the second paragraph, if we do not fix an imaginary unit, then a "complex number" does not have a well-defined imaginary part, but only well-defined up to sign.

Corollary 17.1.7 (page 357, clarification) To avoid a circular argument in constructing the real numbers using condensing sequences, we must formally modify the definition of a condensing sequence so that it refers only to rational numbers: For every positive *rational* number ε, there exists an index N such that if k and $k' \geq N$, then $|a_k - a_{k'}| < \varepsilon$.

Exercise 17.1.6 (page 358, correction) This exercise should be placed in Section 17.2.

In the parenthetical remark, the space $\mathscr{C}(X, \mathbf{R})$ of continuous functions should be replaced by the space of *bounded* continuous functions.

Definition 17.3.1 (page 363, clarification) The term "rapidly decaying" is preferable to "rapidly decreasing." This change will be made if there is a second printing.

Solution to 6.2.3 (page 411, clarification) The phrase "strictly increases with limit 1" means if k is fixed and $m \to \infty$.

Solution to 13.4.3 (page 445, correction) "If $x > 0$, then $-\pi/2 < \theta < \pi/2$, so x and y are both positive, and. . . ." should read "If $x > 0$, then $-\pi/2 < \theta < \pi/2$. If y is also positive, then. . . ."